

Introduction to Portfolio Theory

Prof. Dr. Georg Schlüchtermann

Ludwig-Maximilians-University Munich

(Faculty of Mathematics, Informatics and Statistics) and

University of Applied Sciences Munich

(Department of Mechanical Engineering, Automotive, and Aeronautical Engineering)

November 25, 2007

Summary

The lecture is divided in an introductory macroeconomic and a technical mathematical part. With this the first three chapters present the fundamentals of portfolio theory, like the Bernoulli- resp. the mean-variance principle, the portfolio selection using efficient portfolio lines and the CAPM. While up to now the pure discrete trading models were used one starts with the treatment of time continuous asset models. First the basic pricing models and facts for portfolios are given, like trading strategies and complete markets. Afterwards it is shown that under certain mild assumptions an optimal portfolio selection can be achieved. As decisive tool we give an introduction to optimal control theory in both cases of finite and infinite time horizon for the consumption process. The lecture is completed by a short view in the portfolio theory using derivatives as security instruments.

We need an introductory lecture on mathematical finance for the understanding of this content, since the knowledge on stochastic differential equations is fundamental. Depending how detailed the lecture is presented it would require a course of 3 to 4 hours per week.

Table of Contents

1 Fundamentals of Portfolio Theory - Economic View

1.1 Asset Returns

1.2 Expected Value and Varianz of Assets

1.3 Standard Problem of Asset Allocation

1.3.1 Utility Functions

1.3.2 Bernoulli Principle

1.3.3 The (μ, σ) –Principle

1.3.4 Optimal Decision using the (μ, σ) –Principle

1.4 Diversification

2 The Portfolio Selection Theory - Economic approach

2.1 Fundamentals

2.2 Diversification for the (μ, σ) –principle

2.3 Basic Model of Portfolio Selection

2.3.1 Efficient and Optimal Portfolios

2.3.2 Portfolio Optimization without Riskfree Asset

2.4 Portfolio Optimization with Riskfree Asset

2.4.1 Portfolio Optimization and Separation Theorem of Tobin

2.4.2 Portfolio Risks of Assets

3 The Capital Asset Pricing Model CAPM

3.1 Market Equilibrium

3.2 Hypothesis of the Capital Market Model

3.3 Risk-Return-Relations of certain Assets

3.4 Risk-return-Relations for Portfolios

3.5 Extensions of the CAPM

3.6 Riskadjusted Performance Measures on the Basis of CAPM

3.6.1 Multidimensional Performance

3.6.2 Performance Measure by Sharpe

3.6.3 Performance Measure by Jensen

3.6.4 Comparision of Performance Measures

4 Optimal Portfolios – mathematical approach

4.1 Fundamentals of the Trading strategies

4.2 Portfolio Process and Consumption

4.3 Fundamental Asset Equation

4.4 Complete Market equation

5 Optimal Control

5.1 Formulation of the Stochastic Control Problem

5.2 Utility Functions

5.3 Hamilton-Jacobi-Bellman Equation

5.3.1 Heuristic Derivation

5.4 Rigorous Proof

5.5 Numerical Solutions

6 Optimal Portfolios using Optimal Control Technic

6.1 Optimization under Finite Time Horizon

6.2 Optimization for an Infinite Time Horizon

7 Optimal Portfolios and Derivatives